A First-Repair Mass Zero Without Angular-Moment Cancellation

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Version 0.5 — 31 October 2026

Abstract

We study two weighted histories in a specified outgoing-profile construction. For each fixed source Profile, we prove that every sufficiently small positive angular parameter admits a point strictly inside the first repair window where the mass history vanishes and the angular-weighted history is strictly positive at the same point. The positive parameter threshold may depend on the Profile. The decisive argument compares future moments of two separated repair bumps. At a mass zero, positivity of the second repair coefficient forces the first coefficient to be negative; the decreasing relative weight between the two histories then makes the angular tail negative and the accumulated angular history positive. We give the coefficient, boundary-sign, continuity and coordinate arguments needed to turn this conditional statement into an existence theorem. We further extract a strictly positive angular-history lower bound independent of the selected mass zero, and normalize its bump moment to obtain an explicit cutoff-plateau bound. The corresponding source-bound propositions are formalized in Lean 4 and have a recorded fixed-checkpoint compilation. The result identifies a denominator obstruction for a prospective normalized ratio without requiring that ratio to be introduced into the source construction.

Keywords: weighted moments; localized repair; separated supports; formalized analysis; Lean 4.

1 Introduction

Moment repair can enforce several terminal cancellation conditions while leaving their partial histories markedly different. The location and sign of those partial histories matter when a derived quantity divides by one of them. A terminal identity alone does not determine whether cancellation occurs earlier, or whether a second history cancels at the same point.

This paper analyzes that question for the outgoing-profile construction defined in the pinned `NavierStokesAndEuler` source [1]. The two quantities are the source mass history M and its H-weighted history J. Our initial investigation considered the ratio J/(M H). The principal result is more naturally stated directly in terms of M and J: each fixed source Profile has a positive parameter neighborhood in which every positive parameter admits an interior first-repair point with M=0 and J>0.

The construction prescribes outgoing fields through localized pulse and repair terms. Its two terminal history constraints make partial cancellation a natural question: enforcing both histories at the end of the schedule need not synchronize them inside a repair window. Our analysis concerns that precise feature of the source construction. The names of the histories are used according to their defining integrals, without assigning an additional flux or vorticity interpretation.

The argument has two parts. First, moment comparisons determine the sign of the main-only second repair coefficient. The exact coefficient decomposition and continuity transfer this sign to the actual coefficient near zero. Boundary signs and the intermediate value theorem then

produce an interior mass zero. Second, a relative-weight comparison proves J>0 at every first-window mass zero whose second coefficient is positive, not merely at a particular zero selected by the existence argument.

The mathematical contribution of this paper is the explicit source-linked derivation of this non-simultaneous cancellation and its formalization. The weighted comparison itself uses elementary real analysis. We do not claim priority over an exhaustive mathematical literature; the references establish the construction and formalization context, not a completed novelty survey. The proof and its precise quantifiers are the objects offered for assessment.

Section 2 specifies the construction and notation. Section 3 states the results. Sections 4 and 5 prove the conditional comparison and the existence theorem. Section 6 explains the formal correspondence and recorded computation. Section 7 discusses the interpretation of the result.

2 Construction and notation

2.1 Fixed source Profile

Fix a source `Profile` F of the `NavierStokes.OutgoingProfile` namespace at revision

`8937a8f4cbc7abaab5e9e97d1cc7f5d2319d9538`.

A source Profile contains tail data, a coefficient bound and a reset witness. Its corrected amplitude is the source function `F.amp`; the coefficients contained in its angular-reset witness are not the axial repair coefficients studied below. All these objects belong to the same fixed Profile.

Write c for `F.data.core`. Its parameter structure includes P>0, m>0, 0<λ<1/10 and wait>25. We retain the full source Profile type, including its tail and reset data, rather than substituting an arbitrary amplitude satisfying only selected properties. Let

$$\begin{aligned} B = c.\mathrm{pulseStart}, \qquad L = \frac{13}{\lambda}, \qquad R = e^{L}, \\ a_0 = -\left(\frac{1}{2} + \lambda\right), \qquad a_1 = -\left(\frac{1}{2} + 2\lambda\right) = a_0 - \lambda. \end{aligned} \tag{2.1}$$

The pulse-local radius is r; its source radial coordinate is $X = e^{B} r$. The source's logarithmic coordinate is y=B+log r.

The two repair intervals have endpoints

$$\begin{aligned} l_0 &= e^{L-63/20}, \quad u_0 = e^{L-57/20}, \\ l_1 &= e^{L-23/20}, \quad u_1 = e^{L-17/20}. \end{aligned} \tag{2.2}$$

The parameter bounds imply

$$1 < e^{11/\lambda} < l_0 < u_0 < l_1 < u_1 < R. \tag{2.3}$$

Indeed, L-63/20-11/λ=2/λ-63/20>0, and the remaining comparisons follow by comparing exponential arguments. In particular the first and second repair supports are strictly separated.

2.2 Source histories and pulse-local histories

The source defines the radial fields U_F, E_F and

$$H_F(X,\eta) = \sqrt{2X}\, E_F(X,\eta).$$

Its histories are

$$M_F(\eta,X) = \int_{(0,X]} U_F(u,\eta)\, du, \qquad J_F(\eta,X) = \int_{(0,X]} H_F(u,\eta) U_F(u,\eta)\, du. \tag{2.4}$$

These are the definitions of `Profile.M` and `Profile.J`, respectively [1]. J is an H-weighted history; no identification with vorticity or enstrophy is needed here.

For r≥1 define the local histories

$$\begin{aligned} \mathcal{M}_F(\eta,r) &= \text{massMoment}(c, F.\text{amp}, \eta, B+\log r), \\ \mathcal{J}_F(\eta,r) &= \text{angularMoment}(c, F.\text{amp}, \eta, B+\log r). \end{aligned}$$

The source identities `M_at_exp` and `J_at_exp` apply because B+log r≥0, and give the exact equalities

$$\mathcal{M}_F(\eta,r) = M_F(\eta, e^B r), \qquad \mathcal{J}_F(\eta,r) = J_F(\eta, e^B r). \tag{2.5}$$

There is no additional scale factor in (2.5). The integral prefactors introduced next belong to the histories themselves.

2.3 Repair coefficients and tails

Let $A_F(\eta)$=F.amp(η). It is smooth and strictly positive by the source corrected-amplitude results. Put s(η)=$(1+\eta^2)^{-1}$. Let

$$b_j(x) = \text{LocalizedMomentRepair.bump}(l_j, u_j, x), \qquad j = 0, 1.$$

Each b_j is nonnegative with topological support contained in (l_j,u_j). More precisely, the source bump is supported inside the middle half $[(3l_j+u_j)/4, (l_j+3u_j)/4]$ of its declared window. Positivity throughout the larger window is not an assumption. Define c_j(η) to be component j of the source axial repair coefficient vector

$$\text{LocalizedMomentRepair.coefficients}\big(c.\text{exponents}, c.\text{lower}, c.\text{upper}, \text{debt}(c, A_F, \eta)\big).$$

The pulse is

$$p_F(\eta,x) = A_F(\eta)\, \text{mainPulse}(\lambda \log x) + c_0(\eta) b_0(x) + c_1(\eta) b_1(x). \tag{2.6}$$

For x≥l_0 the main pulse is zero. Thus every future interval starting inside the first repair window sees only the two correction bumps.

Let κ_i=`momentScale c i` and define the strictly positive prefactors

$$K_0(\eta) = \kappa_0 s(\eta), \qquad K_1(\eta) = \kappa_1 s(\eta)^2.$$

The source prefix constants P_M=`prefixM c` and P_J=`prefixJ c` give, for 1≤r≤R,

$$\begin{aligned} \mathcal{M}_F(\eta,r) &= P_M \eta + K_0(\eta) \int_1^r x^{a_0} p_F(\eta,x)\, dx, \\ \mathcal{J}_F(\eta,r) &= P_J \eta s(\eta) + K_1(\eta) \int_1^r x^{a_1} p_F(\eta,x)\, dx. \end{aligned} \tag{2.7}$$

Terminal cancellation, supplied by `pulse_closes_prefix`, changes these to

$$\mathcal{M}_F(\eta, r) = -K_0(\eta) \int_r^R x^{a_0} p_F(\eta, x)\, dx, \qquad \mathcal{J}_F(\eta, r) = -K_1(\eta) \int_r^R x^{a_1} p_F(\eta, x)\, dx. \tag{2.8}$$

For r∈(l_0,u_0), abbreviate

$$T_j(r) = \int_r^R x^{a_0} b_j(x)\, dx, \qquad S_j(r) = \int_r^R x^{a_1} b_j(x)\, dx. \tag{2.9}$$

Then (2.8) becomes

$$\mathcal{M}_F = -K_0(c_0 T_0 + c_1 T_1), \qquad \mathcal{J}_F = -K_1(c_0 S_0 + c_1 S_1). \tag{2.10}$$

All terms in each equation refer to the same F, η and r.

3 Main results

Theorem 3.1 (conditional angular mismatch). Fix a source Profile F. For any real η and any r satisfying l_0<r<u_0,

$$\mathcal{M}_F(\eta, r) = 0 \quad \text{and} \quad c_1(\eta) > 0 \quad \Longrightarrow \quad \mathcal{J}_F(\eta, r) > 0.$$

No sign assumption on η is required in this theorem.

Theorem 3.2 (same-point existence). For every fixed source Profile F there exists ϵ_F>0 such that, for every 0<η<ϵ_F, there exists r∈(l_0,u_0) with

$$M_F(\eta, e^B r) = 0 \quad \text{and} \quad J_F(\eta, e^B r) > 0. \tag{3.1}$$

The positive threshold ϵ_F may depend on F, and the witness radius r may depend on η. Both clauses in (3.1) use that one witness.

Theorem 3.3 (a margin independent of the selected zero). Under exactly the hypotheses of Theorem 3.1, put

$$I_{01} = \int_{-\infty}^{\infty} x^{a_0} b_1(x)\, dx, \qquad G_\lambda = u_0^{-\lambda} - l_1^{-\lambda}.$$

Then

$$0 < K_1(\eta) c_1(\eta) I_{01} G_\lambda \le \mathcal{J}_F(\eta, r). \tag{3.2}$$

The positive product on the left does not depend on which qualifying mass zero r is chosen. It may depend on F and eta. No assumption eta>0, amplitude bound or numerical value of the coefficient is added to this conditional result.

Theorem 3.4 (normalization and a plateau margin). Set $\beta_0 = 1/2 - \lambda$, $\ell = e^{-23/20}$, $v = e^{-17/20}$, and

$$\mu_{01} = \int_\ell^v z^{a_0}\, \mathrm{cutoff}\Big(\frac{z - (\ell + v)/2}{(v - \ell)/4}\Big)\, dz.$$

The exact identities and bound are

$$
\begin{aligned}
I_{01} &= e^{L\beta_0}\mu_{01},\\
\mu_{01} &= e^{-\beta_0}\,\mathrm{rowMoment}(a_0) \geq (v-\ell)/4 > 0,\\
G_\lambda &= e^{-13+23\lambda/20}\big(e^{17\lambda/10}-1\big) > 0.
\end{aligned}
\tag{3.3}
$$

Consequently the hypotheses of Theorem 3.1 also imply

$$
\begin{aligned}
0 &< K_1(\eta)c_1(\eta)e^{L\beta_0}\frac{v-\ell}{4}G_\lambda\\
&\leq \mathcal{J}_F(\eta,r).
\end{aligned}
\tag{3.4}
$$

These are exact-real inequalities. They neither supply an evaluated numerical margin nor require a numerical quadrature of the cutoff for the plateau bound. The notation G_λ is distinct from the negative main-moment difference Δ_M used in Section 5.

4 Proof of the conditional comparison

Lemma 4.1 (tail signs and coefficient sign). At a point satisfying the hypotheses of Theorem 3.1, $T_1>0$, $T_0>0$ and $c_0<0$.

Proof. Since $r<u_0<l_1$ and $u_1<R$, the future interval contains the whole second support. Therefore T_1 equals its whole-line weighted bump moment. The source matrix-entry identity expresses this moment as a positive template row moment times a positive exponential. Hence $T_1>0$. Also $T_0\geq 0$ because its integrand is nonnegative on positive x. The mass-zero identity in (2.10), with $K_0>0$, gives

$$
c_0T_0 = -c_1T_1 < 0. \tag{4.1}
$$

If $T_0=0$, (4.1) is impossible. Thus $T_0>0$, and (4.1) forces $c_0<0$. □

Lemma 4.2 (comparison of relative weights). For every $r\in(l_0,u_0)$,

$$
u_0^{-\lambda}T_0 \leq S_0, \qquad S_1 \leq l_1^{-\lambda}T_1, \qquad l_1^{-\lambda} < u_0^{-\lambda}. \tag{4.2}
$$

Proof. The identity $a_1=a_0-\lambda$ gives $x^{a_1} = x^{a_0}x^{-\lambda}$ for $x>0$. The function $x\mapsto x^{-\lambda}$ strictly decreases since $\lambda>0$. Wherever b_0 is nonzero, $x<u_0$ and thus $x^{-\lambda} \geq u_0^{-\lambda}$. Wherever b_1 is nonzero, $x>l_1$ and $x^{-\lambda} \leq l_1^{-\lambda}$. Multiplying these inequalities by the respective nonnegative integrands $x^{a_0}b_j(x)$ and integrating gives the first two comparisons. The final strict comparison follows from $u_0<l_1$. □

Proof of Theorem 3.1. By Lemma 4.1, $c_0<0$ and $c_1>0$. Consequently the first inequality in (4.2) reverses when multiplied by c_0. Using both tail bounds and then (4.1),

$$
\begin{aligned}
c_0S_0 + c_1S_1 &\leq c_0u_0^{-\lambda}T_0 + c_1l_1^{-\lambda}T_1\\
&= c_1T_1\big(l_1^{-\lambda}-u_0^{-\lambda}\big) < 0.
\end{aligned}
\tag{4.3}
$$

Equation (2.10) and $K_1>0$ now give $\mathcal{J}_F(\eta,r) > 0$. □

The argument explains why moment cancellation is not transported from one weight to the other: a negative first-support contribution receives a larger relative weight than the positive second-support contribution. Positivity of T_0 is a consequence of mass balance, not an additional assumption. It does not imply $b_0(r)>0$ at the zero itself.

4.1 Quantitative and normalized forms

Proof of Theorem 3.3. The support of b_1 is wholly inside (l_1, u_1), and $r < u_0 < l_1 < u_1 < R$. Thus $T_1(r) = I_{01}$. The strict separation and $\lambda > 0$ give $G_\lambda > 0$. Equation (4.3), multiplied by the negative prefactor $-K_1$, reverses its inequality and yields (3.2). Each factor is positive, using $c_1 > 0$ and the source positivity of the full second-bump moment. The factor positivity assertion itself does not require a mass zero. The J inequality does.

Proof of Theorem 3.4. Substitute $x = e^L z$ in the full second-bump integral. The source bump is defined in the radial coordinate, so its endpoints become ℓ and v, and its argument becomes the one in the definition of μ_{01}. The Jacobian and power contribute $e^{L(a_0+1)} = e^{L\beta_0}$. The source matrix-entry identity gives $I_{01} = \mathrm{rowMoment}(a_0)e^{\beta_0(L-1)}$, which proves the second equality in (3.3).

Let $d = (\ell+v)/2$. On the interval $[d-(v-\ell)/8, d+(v-\ell)/8]$ the absolute cutoff argument is at most $1/2$, so the source cutoff equals one. This interval has width $(v-\ell)/4$. Since $0 < z < 1$ and $a_0 < 0$, $z^{a_0} \geq 1$. The integrand is nonnegative on the rest of $[\ell, v]$. Integration proves the plateau bound. Substitution in (3.2), multiplying only by positive factors, gives (3.4). Finally, inserting the explicit endpoints in G_λ and factoring their exponential difference proves the gap identity in (3.3).

The named Lean statements for this argument are listed separately in Appendix C. The displayed rearrangement using $e^t - 1$ is an exact identity, not an error analysis for any floating-point expm1 implementation.

5 Coefficient positivity and interior existence

5.1 Sign of the actual coefficient near zero

Write β_i=a_i+1 and q=L-3. The source main moments and their normalized values are

$$\mu_i = \int_1^R x^{a_i} \mathrm{mainPulse}(\lambda \log x)\, dx, \qquad \mathcal{F}_i = e^{-\beta_i q}\mu_i.$$

Let the positive source template moments and comparison factor be

$$A_i = \mathrm{rowMoment}(a_i), \qquad k = e^{-3\lambda/20}.$$

The correspondence with the source is as follows.

Symbol	Source expression
β_i	`OutgoingPulseBounds.beta c i`
q	`OutgoingPulseBounds.center c 0`
μ_i	`OutgoingSchedule.mainMoment c i`
$\mathcal{F}_i$	`NSRW.P2.normalizedMainMoment c i`
A_i	`OutgoingPulseBounds.rowMoment (c.exponents i)`

Exact locators are the pinned definitions of beta, center and rowMoment, together with the NSRW normalized-moment definition at the execution revision. Thus q is the first repair center in logarithmic coordinates; it is not the endpoint l_0 or u_0. The formalized main-pulse and template comparisons give

$$\mathcal{F}_1 > 0, \qquad \mathcal{F}_0 < k\mathcal{F}_1, \qquad kA_1 \leq A_0. \tag{5.1}$$

For completeness, the strict main comparison first bounds the extra exponential on the short logarithmic interval $0 \le y \le 11/\lambda$ by $e^{-2+3\lambda}$. The condition $\lambda < 1/10$ implies $-2+3\lambda < -3\lambda/20$. Multiplication by the positive normalized first moment provides strictness. The template comparison uses the nonnegative template support in $x > e^{-3/20}$ and the factor x^{λ}. These are separately formalized comparisons [2].

Multiplying (5.1) by the appropriate positive factors yields

$$\mathcal{F}_0 A_1 < k\mathcal{F}_1 A_1 \le \mathcal{F}_1 A_0.$$

Thus

$$\Delta_M = \frac{\mathcal{F}_0}{A_0} - \frac{\mathcal{F}_1}{A_1} < 0. \tag{5.2}$$

The normalized source repair equations identify the main-only second coefficient m_1 as

$$m_1 = \frac{-\Delta_M}{e^{2\beta_0} - e^{2\beta_1}} > 0. \tag{5.3}$$

The denominator is positive because $\beta_0 - \beta_1 = \lambda > 0$. Explicitly, let

$$p_1 = \text{affineCoefficients}(c, 1, 0, 1),$$
$$m_1 = \text{affineCoefficients}(c, 0, 1, 1).$$

Both are constants determined by the fixed core c, independent of η. The actual coefficient of the fixed Profile satisfies the exact decomposition

$$c_1(\eta) = \eta(1 + \eta^2)p_1 + A_F(\eta)m_1. \tag{5.4}$$

In particular $c_1(0) = A_F(0)m_1 > 0$. The corrected amplitude is smooth, so (5.4) is continuous in η. The preimage of the positive half-line is an open neighborhood of zero. It therefore contains $(-\epsilon_F, \epsilon_F)$ for some $\epsilon_F > 0$. Restricting to $0 < \eta < \epsilon_F$ supplies the required sign $c_1(\eta) > 0$.

5.2 Boundary signs and continuity

Lemma 5.1 (first-window boundary signs). If $\eta > 0$ and $c_1(\eta) > 0$, then

$$\mathcal{M}_F(\eta, l_0) > 0, \qquad \mathcal{M}_F(\eta, u_0) < 0. \tag{5.5}$$

Proof. The source prefix constant satisfies

$$P_M = 4 + \int_0^B e^y \,\text{dropCoefficient}(c.m, y)\, dy > 0,$$

because the integrand is nonnegative. Both repair bumps vanish before and at l_0, so the partial pulse integral in (2.7) at l_0 is nonnegative: $A_F > 0$, the main pulse is nonnegative for nonnegative argument, and $x^{a_0} > 0$ on $x \ge 1$. The term $P_M\eta$ is strictly positive.

At u_0, the main pulse and first bump vanish on the future interval. The mass-tail identity reduces to

$$\mathcal{M}_F(\eta, u_0) = -K_0(\eta)c_1(\eta) \int_{u_0}^R x^{a_0} b_1(x)\, dx < 0,$$

since the second-bump moment is strictly positive. □

Lemma 5.2 (continuity on the repair interval). For fixed F and η, the map r↦$\mathcal{M}_F(\eta, r)$ is continuous on [l_0,u_0].

Proof. The source logarithmic mass primitive is

$$4\eta + \int_0^y e^t \operatorname{axial}(c, A_F)(t, \eta)\, dt.$$

Its integrand is continuous, so it is continuous in y. The map r↦B+log r is continuous on [l_0,u_0], where r>0. Composition proves the assertion. □

Proof of Theorem 3.2. Choose ϵ_F from (5.4) and continuity. For every $0<\eta<\epsilon_F$, Lemmas 5.1 and 5.2 and the intermediate value theorem give a zero in [l_0,u_0]. The strict signs in (5.5) exclude both endpoints, so the zero belongs to (l_0,u_0). Theorem 3.1 gives positive local angular history at that very zero. Equation (2.5) transfers both statements to the same source radial coordinate $e^B r$. □

6 Formalization and computational record

The proofs use Lean 4 [3] with the source and dependencies fixed by the recorded environment. The theorem-to-declaration correspondence is given in Appendix A. The final declaration binds one F and one η, obtains a single radius from the conditional intermediate-value theorem, and passes that radius to the angular-positivity theorem. It does not combine separately chosen existential witnesses.

The recorded compilation at NSRW revision `3a50b5f50b034319a469b9de68f556be70ff f02c` used Lean 4.34.0-rc2 and the source revision of Section 2. The 12 October 2026 record comprises a source-target build and fourteen fresh external modules. All fifteen stages exited zero with empty stderr. Nineteen reported axiom surfaces for the final local, source-facing and composition declarations contain `propext`, `Classical.choice` and `Quot.sound`, with no `sorryAx` [4, 5]. A retained nonfatal stdout linter warning is documented in those records.

The formal compilation establishes the stated Lean propositions relative to the imported definitions and proof environment. The prose proof above explains their mathematical mechanism. The execution reused the pinned upstream cache; it was not a fresh rebuild of the entire upstream library. Execution and archive review were carried out within one operator trust domain. These facts describe the computational record, not an external peer-review process.

The quantitative extension has a separate proof checkpoint `fba27381bcfda3af6f03e2fbae 82f6db016ca534` and replay checkpoint `7f29ebf0e2b894bf598144628230ae09df436c3d`. Its retained October 22 execution records six compilation stages, five fresh external modules and ten named quantitative/normalization axiom surfaces, with stage exit zero, empty stderr and the same standard three axioms [6]. It is distinct from the October 12 run; counts and checkpoints are not combined. The dated receipt admits the ten propositions within the same operator trust domain. No new Lean execution was performed to prepare this manuscript revision.

7 Interpretation and further questions

The result distinguishes terminal cancellation from intermediate behavior. M and J can both satisfy prescribed endpoint cancellation while having different signs or zeros inside a repair interval. The proof identifies the responsible structure: opposite-signed repair coefficients, separated supports and a strictly decreasing relative weight.

For a prospective real ratio Q=J/(M H), the point in Theorem 3.2 fails the condition M H≠0 regardless of H at that point. This is a domain obstruction. It does not supply a quotient limit,

divergence rate or singularity classification. In particular, total real division in Lean should not be confused with a partial-domain ratio used in analysis.

The existence theorem remains existential even after the exact quantitative extension. It does not compute ϵ_F or r. Stronger zero geometry calls for a derivative argument and localization inside the actual positive region of the first bump, rather than positivity throughout its larger declared window. In particular, the positive integral T_0 alone does not establish positive bump density at the witness. These observations identify a concrete route toward investigating monotonicity, uniqueness and crossing behavior; those statements are not added to the present formal result. The full zero set outside the positive parameter neighborhood is another open question.

8 Conclusion

In the fixed outgoing-profile construction, an interior mass cancellation is accompanied by strictly positive angular-weighted history whenever the second repair coefficient is positive. Moment comparison, coefficient continuity and boundary signs ensure that such a point exists for every sufficiently small positive parameter of each fixed source Profile. The proof exposes a concrete mechanism for non-simultaneous cancellation and connects that mechanism to exact source-facing formal propositions. The quantitative extension gives a lower bound for the local angular history at every qualifying mass zero, with an exact fixed-coordinate normalization and a conservative plateau estimate.

Code availability and authorship

The formal input files and replay runner are identified by NSRW revision `3a50b5f50b034319a469b9de68f556be70fff02c` [2]. The two post-execution records [4, 5] are separate documents; their text is not asserted to exist at the execution revision. The accompanying reproduction guide identifies the selected inputs and receipt snapshots supplied with the selected reproduction distribution. Original raw execution evidence is privately retained. The software is archived under DOI 10.5281/zenodo.23381778. Software release v0.4.1 contains the original selected reproduction asset and P01 v0.4; it predates the quantitative extension and does not contain the P1 inputs. The P1 proofs and runner are fixed at their separate October 22 replay revision. This v0.5 manuscript includes the quantitative extension. The earlier v0.4 files and their checksums are preserved separately; the manuscript and software use distinct version numbers.

Yun Kwansub is the named human author. AI assistance was used in analysis, formal-code development and manuscript preparation. The source construction and imported formal mathematics are attributed to their source repositories and authors.

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References

[1] OpenAI and repository contributors. *NavierStokesAndEuler*. Source revision `8937a8f4cbc7abaab5e9e97d1cc7f5d2319d9538`, especially `OutgoingSchedule.lean`, `LocalizedMomentRepair.lean`, `OutgoingPulseBounds.lean` and `OutgoingProfile.lean`. Fixed source.

[2] NSRW repository. *NavierStokes Research Workbench*: formal inputs and replay runner at revision `3a50b5f50b034319a469b9de68f556be70fff02c`. Fixed repository.

[3] Leonardo de Moura and Sebastian Ullrich. The Lean 4 Theorem Prover and Programming Language. In *Automated Deduction—CADE 28*, LNCS 12699, 2021. doi:10.1007/978-3-030-79876-5_37.

[4] NSRW. *P2 Z-core Colab execution receipt*, 12 October 2026. Local document.

[5] NSRW. *P2 Z+J Colab execution receipt*, 12 October 2026. Local document.

[6] NSRW. *P1 quantitative-margin and normalization receipt*, 22 October 2026. Fixed receipt. Fixed P1 inputs.

A Formal correspondence

Module filenames and local navigation are given below. All listed proof modules are also available under `formal/p2/` at the fixed execution revision [2].

Mathematical statement	**Lean declaration in `NSRW.P2`**	**Module**
Local mass partial/tail identities	`localMass_partial_identity`, `localMass_tail_identity`	`P2_Z01_LocalMassIdentities.lean`
Local angular partial/tail identities	`localAngularMoment_partial_identity`, `localAngularMoment_tail_identity`	`P2_ZPLUS_J_FirstRepairAngularMoment.lean`
Strict support separation	`first_upper_lt_second_lower`	`P2_ZPLUS_J_FirstRepairAngularMoment.lean`
Future mass balance	`firstRepair_mass_zero_tail_balance`	`P2_ZPLUS_J_FirstRepairAngularMoment.lean`
$T_0>0$ and $c_0<0$	`firstBumpTail_pos_and_firstCoeff_neg`	`P2_ZPLUS_J_FirstRepairAngularMoment.lean`
Relative-weight inequalities	`firstBump_angularTail_lower`, `secondBump_angularTail_upper`	`P2_ZPLUS_J_FirstRepairAngularMoment.lean`
Theorem 3.1	`angularMoment_pos_of_firstRepair_mass_zero`	`P2_ZPLUS_J_FirstRepairAngularMoment.lean`
Coefficient positivity near zero	`actualSecondRepairCoefficient_pos_small_positive`	`P2_C04_SmallPositiveEta.lean`
Strict boundary signs	`localMass_lower_pos`, `localMass_upper_neg_of_secondCoeff_pos`	`P2_Z02_BoundaryMassSigns.lean`
Conditional interior mass zero	`exists_firstRepair_zero_of_secondCoeff_pos`	`P2_Z03_ConditionalInteriorZero.lean`
Source mass coordinate bridge	`localMass_eq_profileM`	`P2_Z04_FirstRepairInteriorZero.lean`
Source angular coordinate bridge	`localAngularMoment_eq_profileJ`	`P2_ZPLUS_J_SourceComposition.lean`
Theorem 3.2	`profileMJ_firstRepair_zero_small_positive`	`P2_ZPLUS_J_SourceComposition.lean`

B Reproduction identity

Item	Recorded identity
NSRW execution revision	`3a50b5f50b034319a469b9de68f556be70fff02c`
Upstream revision	`8937a8f4cbc7abaab5e9e97d1cc7f5d2319d9538`
Dependency manifest Git blob	`f07a8454cb6200d90bcc4371bc9965e9f8f46c7d`
Lean	`4.34.0-rc2`
Replay runner	`scripts/run-p2-z-replay.py`
Reference execution	`nsrw-z-phase-replay-20261012T065216884451Z`

The runner requires the pinned source environment and exact NSRW checkout. It does not install dependencies. A new execution will have its own logs, timings and generated artifact hashes. Detailed invocation and distributed-evidence information is in the reproduction guide and the two records [4, 5].

C Quantitative correspondence and hypothesis boundaries

All ten declarations below are in `NSRW.P2`, at the P1 replay revision [6]. The first four belong to `P2_ZPLUS_J_QuantitativeMargin.lean`; the final six belong to `P2_ZPLUS_J_MarginNormalization.lean`.

Declaration	Additional hypotheses beyond the typed source object	Content
`secondBump_massTail_eq_full`	r<upper(0) only	T1 equals its full second-bump integral
`firstRepair_weightGap_pos`	None beyond core Parameters	$G_\lambda > 0$
`angularMoment_ge_firstRepair_margin`	Same F,eta,r; l0<r<u0; local M=0; c1>0	Product at most local J in (3.2)
`firstRepair_margin_pos`	c1>0 only, no r or mass-zero assumption	Strict positivity of that product
`firstRepair_weightGap_exp_eq`	None beyond core Parameters	Exact gap factorization
`normalizedSecondBumpMoment_eq_interval`	None beyond core Parameters	Whole-line normalized moment equals its interval integral
`secondBumpWeightedMoment_eq_normalized`	None beyond core Parameters	$I_{01} = e^{L\beta_0}\mu_{01}$
`normalizedSecondBumpMoment_eq_rowMoment`	None beyond core Parameters	$\mu_{01} = e^{-\beta_0}$ rowMoment(a_0)
`normalizedSecondBumpMoment_ge_plateau`	None beyond core Parameters	Plateau bound in (3.3)
`angularMoment_ge_plateau_margin`	Same four extra conditions as the first J inequality	Local J bound in (3.4)

The new quantitative declarations conclude bounds for the local angular history. Equation (2.5) provides the existing coordinate equality; this revision does not report a newly compiled named quantitative Profile.J wrapper. A source amplitude estimate, numerical q>0, a computable Profile and a root finder are separate tasks.